Math 0350 Practice Test **Unit Three**

Chapters 10 part II and Chapter 11 **Martin-Gay 4th edition**

Rationalize each denominator. Assume all variables are positive.

1. $\frac{7\sqrt{3}}{\sqrt{7}}$ **Solution:** $\frac{7\sqrt{3}}{\sqrt{7}}\cdot\frac{\sqrt{7}}{\sqrt{7}}=\frac{7\sqrt{3}\cdot\sqrt{7}}{7}=\frac{\cancel{7}\sqrt{21}}{\cancel{7}}=\boxed{\sqrt{21}}$

2. $\sqrt{\frac{5}{12}}$ **Solution:** $\sqrt{\frac{5}{12}}=\sqrt{\frac{5}{12}\cdot\frac{12}{12}}=\frac{\sqrt{5\cdot3\cdot2^2}}{12}=\frac{\cancel{2}\sqrt{5\cdot3}}{\cancel{12}}=\frac{\sqrt{15}}{6}$

3. $\frac{3}{\sqrt{27x}}$ **Solution:** $\frac{3}{\sqrt{27x}}\cdot\frac{\sqrt{27x}}{\sqrt{27x}}=\frac{3\sqrt{3\cdot3\cdot3x}}{27x}=\frac{\cancel{3}\cdot\cancel{3}\sqrt{3x}}{\cancel{3}\cdot\cancel{3}\cdot3x}=\boxed{\frac{\sqrt{3x}}{3x}}$

Solve each of the following for x:

4. $\sqrt{x+7}=x+5$ **Solution:**

$\left(\sqrt{x+7}\right)^2=(x+5)^2$ square both sides

$x+7=(x+5)(x+5)$ FOIL

$x+7=x^2+10x+25$ subtract x and 7 from both sides

$0=x^2+10x+25-x-7$

$0=x^2+9x+18$

This is a quadratic function. We can solve using any method that we know. I will use factoring:

$x^2+9x+18=0$

$(x+6)(x+3)=0$

$x+6=0 \qquad x+3=0$

$x=-6 \qquad x=-3$

Always check the radical equations. Very often the solutions are solutions of the quadratic function, but are not solution of the radical (because the some values make the function under the radical negative and make the solution invalid!)

Check:

$x=-6$

$\sqrt{x+7}=x+5$

$\sqrt{-6+7}=-6+5$

$\sqrt{1}=-1$

$1\neq-1 \qquad x\neq-6$

extraneous solution

$x=-3$

$\sqrt{x+7}=x+5$

$\sqrt{-3+7}=-3+5$

$\sqrt{4}=2$

$2=2 \qquad \boxed{x=-3}$

I ALWAYS check my solutions if I square both sides, because I may have added solutions by squaring. Example: $\left(\sqrt{-2}\right)^2 = -2$, but $\sqrt{-2}$ is not defined to begin with. We do not usually take square root of negative values. Square root is always a positive number, and a square of a number is a positive number.

5. $\sqrt[3]{2x-3} - 1 = 2$

Solution:

$\sqrt[3]{2x-3} - 1 + 1 = 2 + 1$

$\sqrt[3]{2x-3} = 3$ power both sides by 3

$\left(\sqrt[3]{2x-3}\right)^3 = 3^3$

$2x - 3 = 27$

$2x - 3 + 3 = 27 + 3$

$2x = 30$ divide both sides by 2

$x = 15$

Check:

$\sqrt[3]{2x-3} - 1 = 2$

$\sqrt[3]{2(15)-3} - 1 = 2$

$\sqrt[3]{30-3} - 1 = 2$

$\sqrt[3]{27} - 1 = 2$

$3 - 1 = 2$

$2 = 2$ $\boxed{x = 15}$

6. $\sqrt{x+5} = 1 + \sqrt{x}$

Solution:

$\left(\sqrt{x+5}\right)^2 = \left(1+\sqrt{x}\right)^2$ square both sides

$x + 5 = \left(1+\sqrt{x}\right)\left(1+\sqrt{x}\right)$ FOIL

$x + 5 = 1 + 2\sqrt{x} + x$ subtract x and 1 from both sides

$\cancel{x} + 5 \cancel{-x} - 1 = \cancel{1} + 2\sqrt{x} \cancel{+x} \cancel{-x} \cancel{-1}$

$4 = 2\sqrt{x}$ divide both sides by 2

$2 = \sqrt{x}$ square both sides

$2^2 = \left(\sqrt{x}\right)^2$

$4 = x$

Check:

$\sqrt{x+5} = 1 + \sqrt{x};$ $x = 4:$ $\sqrt{9} = 1 + 2$

$\sqrt{4+5} = 1 + \sqrt{4}$ $3 = 3$ $\boxed{x = 4}$

7. In right triangle ABC, angle C is $90°$. Side a is 4 inches and side b is 6 inches. Find the length of the hypotenuse, side c. Give the answer both in simple radical form and rounded off to 2 decimal places.

Solution:

$a^2+b^2=c^2$

$4^2+6^2=c^2$

$16+36=c^2$

$52=c^2$ take square root of both sides

$\sqrt{52}=\sqrt{c^2}$

$c=\sqrt{52}=\boxed{\sqrt{4\cdot 13}=2\sqrt{13}\approx 7.21}$ inches

8. Write $\sqrt{-64}$ in terms of *i*.

Solution: Minus sign under the radical is a NO NO thing! So the first step is to remove the minus, and replacing it with i **outside** of the radical!

$\sqrt{-64}=i\sqrt{64}=i\cdot 8=\boxed{8i}$

9. Multiply: $\sqrt{-12}\cdot\sqrt{-3}$.

Solution:

$\sqrt{-12}\cdot\sqrt{-3}=i\sqrt{12}\cdot i\sqrt{3}=$

$=i^2\sqrt{12}\cdot\sqrt{3}=(-1)\sqrt{12\cdot 3}=$

$=-\sqrt{2\cdot 2\cdot 3\cdot 3}=-2\cdot 3=\boxed{-6}$

10. Divide: $\dfrac{\sqrt{-16}}{\sqrt{4}}$

Solution: $\dfrac{\sqrt{-16}}{\sqrt{4}}=\dfrac{i\sqrt{16}}{\sqrt{4}}=\dfrac{4i}{2}=\boxed{2i}$

Perform the indicated operations. Leave your final answer in $a+bi$ form.

11. $(-2+4i)(5-6i))$

Solution: $(-2+4i)(5-6i)=$

$=-2\cdot 5+4i\cdot 5-2(-6i)+4i(-6i)=$

$=-10+20i+12i-24i^2=-10+32i-24(-1)=$

$=-10+32i+24=\boxed{14+32i}$

12. $\dfrac{-3+2i}{1-i}$

Solution: Multiply the denominator and the numerator with the conjugate of the denominator.

$\dfrac{-3+2i}{1-i}\cdot\dfrac{1+i}{1+i}=\dfrac{(-3+2i)(1+i)}{(1-i)(1+i)}$ FOIL (continues next page)

$$=\frac{-3(1)+2i(1)-3i+2i(i)}{1(1)-i(1)+1(i)-i(i)}=$$

$$=\frac{-3+2i-3i+2i^2}{1\cancel{-i}\,\cancel{+i}-i^2}=$$

$$=\frac{-3-i+2(-1)}{1-(-1)}=$$

$$=\frac{-3-i-2}{1+1}=\frac{-5-i}{2}=\boxed{-\frac{5}{2}-\frac{1}{2}i}$$

13. $(7+5i)-(3-4i)$ **Solution:** $(7+5i)-(3-4i)=7+5i-3+4i=\boxed{4+9i}$

14. $(3+4i)^2$ **Solution:** $(3+4i)^2=(3+4i)(3+4i)=$ FOIL

$=3\cdot 3+4i\cdot 3+3\cdot 4i+4i\cdot 4i=$

$=9+12i+12i+16i^2=$

$=9+24i+16(-1)=$

$=9+24i-16=\boxed{-7+24i}$

Solve using the square root property:

15. $2x^2-40=0$ **Solution:** $2x^2=40$ divide both sides by 2

$x^2=20$ take square root of both sides

$x=\pm\sqrt{20}$ simplify

$\boxed{x=\pm\sqrt{4\cdot 5}=\pm 2\sqrt{5}}$

Square root of 20 is $\sqrt{20}$ only, but $\sqrt{x^2}=|x|=\pm x$, depending of weather x is a positive, or negative number. The square root function yields a positive value! If $x = 2$, for example, $|x| = 2$, but if $x = -2$, the answer is $|x| = -(-2) = 2$.

When taking square root of an equation we ALWAYS have 2 solutions.

16. $(x+7)^2=-49$ **Solution:** $(x+7)^2=-49$ take square root of both sides

$\sqrt{(x+7)^2}=\sqrt{-49}$

$\pm(x+7)=i\sqrt{49}$

$x+7=\pm i\sqrt{49}$

$x+7=\pm 7i$

$\boxed{x=-7\pm 7i}$

Do not forget that even $\sqrt{49}=7$, $\sqrt{(x+7)^2}=|x+7|=\pm(x+7)$, so there are 2 solutions of this problem!!!

Solve by completing the square:

17. $x^2+8x=2$

Solution: To complete the square one should add $\left(\frac{8}{2}\right)^2$ to both sides of the equation. (This is the coefficient of the term containing *x*, divided by *2* and squared.)

$x^2+8x+\left(\frac{8}{2}\right)^2=2+\left(\frac{8}{2}\right)^2$

$x^2+8x+4^2=2+4^2$

$(x+4)^2=18$ take square root of both sides

$\sqrt{(x+4)^2}=\sqrt{18}$

$x+4=\pm\sqrt{18}$

$x=-4\pm\sqrt{9\cdot 2}$

$\boxed{x=-4\pm 3\sqrt{2}}$

18. $x^2-2x+10=0$

Solution: To complete the square one should subtract 10 from both sides (to avoid confusion) and add $\left(\frac{-2}{2}\right)^2$ to both sides of the equation. (This is the coefficient of the term containing *x*, divided by *2* and squared.)

$x^2-2x+10=0$

$x^2-2x+\left(\frac{-2}{2}\right)^2=-10+\left(\frac{-2}{2}\right)^2$

$x^2-2x+(-1)^2=-10+(-1)^2$

$x^2-2x+1=-10+1$

$(x-1)^2=-9$ take square root of both sides

$\sqrt{(x-1)^2}=\sqrt{-9}$ remove the "minus sign" immediatelly

$x-1=\pm i\sqrt{9}$

$x-1=\pm 3i$

$\boxed{x=1\pm 3i}$

Solve using the quadratic formula: $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

19. $9x^2-10x+2=0$

Solution: $a=9, b=-10, c=2$

$$x=\frac{-(-10)\pm\sqrt{(-10)^2-4(9)(2)}}{2(9)}x=\frac{10\pm\sqrt{100-72}}{18}=\frac{10\pm\sqrt{28}}{18}=\frac{10\pm 2\sqrt{7}}{18}=\boxed{\frac{5\pm\sqrt{7}}{9}}$$

Don't forget to divide by 2 ALL the terms in the rational answer!!!

20. $x^2+2x+17=0$

Solution: $a=1, b=2, c=17$

$$x=\frac{-2\pm\sqrt{2^2-4(1)(17)}}{2(1)}=\frac{-2\pm\sqrt{4-68}}{2}=\frac{-2\pm\sqrt{-64}}{2}$$

$$x=\frac{-2\pm 8i}{2}=\boxed{-1\pm 4i}$$

21. $x^2+20=7x$

Solution: Move all terms to one side: $x^2-7x+20=0$

$a=1, b=-7, c=20$

$$x=\frac{-(-7)\pm\sqrt{(-7)^2-4(1)(20)}}{2(1)}=\frac{7\pm\sqrt{49-80}}{2}=$$

$$=\frac{7\pm\sqrt{-31}}{2}=\frac{7\pm i\sqrt{31}}{2}=\boxed{\frac{7}{2}\pm\frac{\sqrt{31}}{2}i}$$

Solve using any method:

22. $4x^4-5x^2+1=0$

Solution: Let $x^2=y$. Then the equation would look way more simpler because $x^4=\left(x^2\right)^2=y^2$, so:

$4y^2-5y+1=0$

This is a regular quadratic equation. You can solve it with any method you know. I am going to use factorization:

$(4y-1)(y-1)=0$

$4y-1=0 \qquad y-1=0$

$4y=1 \qquad y=1$

$y=\frac{1}{4}$

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You found *y*, but the question required you to find the values for *x*. Recall that $x^2=y$:

$y=\frac{1}{4} \qquad y=1$

$x^2=y=\frac{1}{4} \qquad x^2=y=1$

$x^2=\frac{1}{4} \qquad x^2=1$ Take square root of both sides

$\boxed{x=\pm\frac{1}{2} \qquad x=\pm 1}$ Remember: equation with a 4 power has 4 solutions!

23. $x+7\sqrt{x}-8=0$

Solution: Let $\sqrt{x}=y$. Then the equation would look way more simpler because $x=\left(\sqrt{x}\right)^2=y^2$, so:

$$y^2+7y-8=0$$
$$(y+8)(y-1)=0$$
$$y+8=0 \qquad y-1=0$$
$$y=-8 \qquad y=1$$
$$\sqrt{x}=y=-8 \qquad \sqrt{x}=y=1$$
$$\sqrt{x}\neq -8 \qquad \sqrt{x}=1 \quad \text{square both sides}$$
$$\boxed{x=1}$$

24. $2x^{\frac{2}{3}}-3x^{\frac{1}{3}}+1=0$

Solution: Let $x^{\frac{1}{3}}=y$. Then $x^{\frac{2}{3}}=\left(x^{\frac{1}{3}}\right)^2=y^2$, so:

$$2y^2-3y+1=0$$
$$(2y-1)(y-1)=0$$
$$2y-1=0 \qquad y-1=0$$
$$2y=1 \qquad y=1$$
$$y=\frac{1}{2}$$
$$x^{\frac{1}{3}}=y=\frac{1}{2} \qquad x^{\frac{1}{3}}=y=1$$
$$x^{\frac{1}{3}}=\frac{1}{2} \qquad x^{\frac{1}{3}}=1 \quad \text{power both sides by 3}$$
$$\left(x^{\frac{1}{3}}\right)^3=\left(\frac{1}{2}\right)^3 \qquad \left(x^{\frac{1}{3}}\right)^3=1^3$$
$$\boxed{x=\frac{1}{8} \qquad x=1}$$

25. $2x-1=x^2$

Solution: Subtract 2x and add 1 to both sides:

$$\cancel{2x}\cancel{-1}\cancel{-2x}\cancel{+1}=x^2-2x+1$$
$$0=x^2-2x+1$$

This is a quadratic function. Solve by using any method:

$$0=(x-1)(x-1)$$
$$0=x-1$$
$$\boxed{x=1}$$

26. $\frac{2}{x-1}-\frac{3}{x+1}=1$

Solution: Find the least common denominator and multiply both sides of the equation with the LCD:

$$\frac{2}{x-1}-\frac{3}{x+1}=1$$
$$LCD=(x-1)(x+1)$$
$$\left(\frac{2}{x-1}-\frac{3}{x+1}\right)(x-1)(x+1)=1(x-1)(x+1)$$
$$\frac{2}{\cancel{x-1}}\cancel{(x-1)}(x+1)-\frac{3}{\cancel{x+1}}(x-1)\cancel{(x+1)}=1(x-1)(x+1)$$
$$2(x+1)-3(x-1)=1(x-1)(x+1)$$
$$2x+2-3x+3=x^2-1$$

Simplify and move all the terms to the one side of the equation:

$$-x+5=x^2-1$$
$$\cancel{-x}\cancel{+5}\cancel{+x}\cancel{-5}=x^2-1+x-5$$
$$0=x^2+x-6$$
$$0=(x-2)(x+3)$$
$$x-2=0 \qquad x+3=0$$
$$\boxed{x=2 \qquad x=-3}$$

Always check if the solutions do not make the denominator 0. Division by 0 is IMPOSSIBLE!

27. $(2x-1)^2-6(2x-1)+5=0$

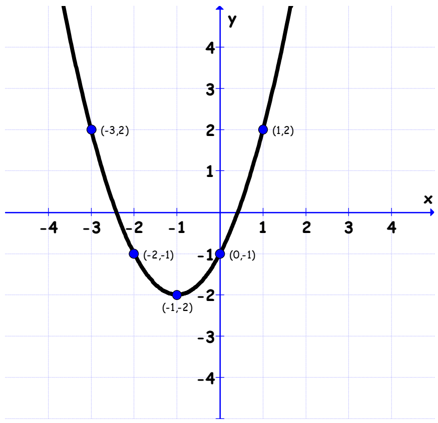
Solution: Let $2x-1=y$. Then $(2x-1)^2=y^2$, so:

$$y^2-6y+5=0$$
$$(y-5)(y-1)=0$$
$$y-5=0 \qquad y-1=0$$
$$y=5 \qquad y=1$$
$$2x-1=y=5 \qquad 2x-1=y=1$$
$$2x-1=5 \qquad 2x-1=1$$
$$2x=6 \qquad 2x=2$$
$$\boxed{x=3 \qquad x=1}$$

Sketch a graph of each of the following quadratic functions. Label the vertex and 2 other exact points. List the domain and range.

28. $f(x)=(x+1)^2-2$

Solution: To find the points, first find the value for x at the vertex and then pick 2 values to the right and to values to the left. Substitute in the function and calculate the y value at these points. The vertex for a equation $f(x)=(x-h)^2+k$ is (h,k), so for this equation it is at $(-1,-2)$.

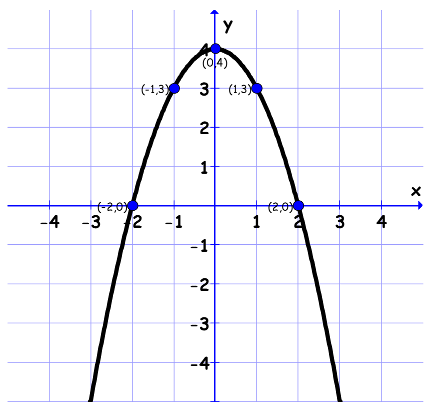


Domain: $(-\infty,+\infty)$

Range: $[-2,+\infty)$

29. $f(x)=4-x^2$

Solution:



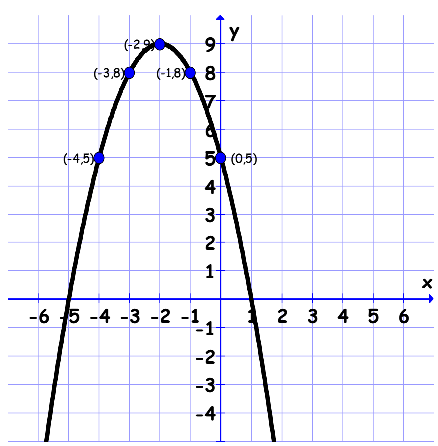
Domain: $(-\infty,+\infty)$

Range: $(-\infty,4]$

30. $f(x)=-x^2-4x+5$

Solution: $x_{vertex}=-\dfrac{b}{2a}=-\dfrac{-4}{2(-1)}=\dfrac{4}{-2}=-2$

$y_{vertex}=f(-2)=-(-2)^2-4(-2)+5=-4+8+5=9$



Domain: $(-\infty,+\infty)$

Range: $(-\infty,9]$